BEARING CAPACITY OF SURFACE WEDGE SHAPED FOUNDATIONS

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Summary

A two dimensional problem of bearing capacity of surface wedge shaped foundation with different semi angles $\beta$ and roughness on homogeneous soil and subjected to axial load is analyzed on the basis of plastic theory. The soil is considered as a perfectly rigid plastic material obeying the Mohr–Coulomb failure criterion. The results of analyses are presented as bearing capacity factors $N_c$, $N_q$, and $N_r$ in terms of angle of internal friction $\phi$ of soil, semi angle $\beta$ and roughness of the foundation.

An experimental investigation was made to obtain bearing capacity factors for estimating the ultimate bearing capacity of wedge shaped foundation with various semi angles $\beta$ and roughness on clay and sand. Soft saturated Ariake clay and dry Toyoura sand were used in these tests. Reasonable agreement was found between the experimental and the theoretical bearing capacity of wedge shaped foundations.

Key Words: bearing capacity, clay, model test, sand, surface foundation

INTRODUCTION

Every civil engineering structure must have a proper foundation. Foundation is that part of the structure which is in direct contact with and transmitting loads to the ground. The supporting power of soil is referred to as its bearing capacity. The bearing capacity of surface wedge shaped foundations under axial load, with different semi angle $\beta$ and roughness can generally be estimated with sufficient accuracy based on a two dimensional analysis using plastic theory. The most important parameters for calculating the ultimate bearing capacity of the foundation are the bearing capacity factors. In this paper, the bearing capacity factors were calculated by limit equilibrium method. The slip line method was used for the calculation of the slip line net near the wedge base with various semi angles $\beta$ and roughness.$^{1, 2}$

An experimental investigation was made of the bearing capacity factors for estimating the ultimate bearing capacity of shallow wedge foundation on clay and sand under axial loads.
THEORY

The components \( \sigma_1 \) and \( \sigma_3 \) are principal stresses in the \((x,y)\) plane and \( \sigma_1 > \sigma_3 \). The principal stresses are obtained from the components of stress \( \sigma_x, \sigma_y, \tau_{xy} \) according to the following equation.

\[
\frac{\sigma_1}{\sigma_3} = \frac{1}{2} (\sigma_x + \sigma_y) \pm \left[ \frac{1}{4} (\sigma_x - \sigma_y)^2 + \tau_{xy}^2 \right]^{1/2}
\]

(1)

The resulting stress components satisfying the equations of equilibrium in two dimensions are

\[
\begin{align*}
\frac{d\sigma_x}{dx} + \frac{d\tau_{xy}}{dy} = & \gamma \\
\frac{d\tau_{xy}}{dx} + \frac{d\sigma_y}{dy} = & 0
\end{align*}
\]

(2)

where \( \gamma \) is the unit weight of soil.

The Mohr-Coulomb failure criteria requires the following equation to be satisfied.

\[
\sqrt{\frac{1}{4} (\sigma_x - \sigma_y)^2 + \tau_{xy}^2} - \left( \frac{\sigma_x + \sigma_y}{2} \sin \phi + C \cos \phi \right) = 0
\]

(3)

Combining equation (2) and (3) the slip line field around the wedge can be developed. The general formula of the ultimate bearing capacity can be represented by

\[
q_u = cN_c + \gamma DN_a + \gamma B \frac{N_r}{2}
\]

(4)

as summation process of some equations

where \( q_u \) is the ultimate bearing capacity

\( N_c, N_a, N_r \) are bearing capacity factors

\( c = \text{Cohesion}, D = \text{Depth of foundation} \)

\( B = \text{Width of wedge} \)

From the above basic equation (eq.4) ultimate bearing capacity factors have been computed

For clay \((\phi=0, N_r=0, N_a=1)\)

\[
q_u = N_c + \frac{\gamma D}{c}
\]

(5)

For sand \((c=0)\)

\[
\frac{q_u}{\gamma B} = \frac{N_r}{2} + \frac{D}{B} N_a
\]

(6)

MODEL TESTS

Experimental investigation of surface wedge shaped foundations of different semi angles \( \beta \) and roughness was conducted. Wedge footings of length 16cm and width 4 cm with various semi angles \( \beta = 15^\circ, 30^\circ, 45^\circ, 60^\circ \) and \( 90^\circ \) were used for this experiment. A semi cylindrical footing of radius 2cm and length 16cm was also used. The material of wedge footings was acrylic having a modulus of elasticity of \( 0.3 \times 10^4 \) MPa.

The wedges were tested with smooth and fully rough surfaces. It was made rough by
gluing sand paper on the contact area with the soil. An LVDT (Linear Voltage Differential Transducer) was used to measure the settlements, and a load cell was used to measure the axial loads. The applied load and the settlements were recorded by a data logger. The sand was rained from a height of 30 cm in a wooden box, 40 cm × 30 cm × 30 cm having a total volume of 28485.94 cm³ to obtain a constant density of 1.35 g/cm³. Higher densities of γ equal to 1.45 g/cm³ and 1.55 g/cm³ have been obtained by compacting sand in 10 cm thick layers. The angle of internal friction φ at different densities was obtained from constant volume direct shear tests. Soft saturated Ariake clay whose remolded undrained shear strength Cu = 1.5 kPa, was packed into the wooden box. The wedge shaped foundation is schematically presented in Fig. 1.

Fig. 1 Cross section of wedge shaped foundation.

Fig. 2 Bearing capacity factor (a) Na, (b) Nr, (c) Nc with semi angle β, angle of internal friction Φ.
Fig. 3 Load settlement curves of wedge foundations on (a) loose sand (b) medium sand (c) dense sand.

Fig. 4 Load settlement curves of wedge foundations for different semi angle $\beta$ on clay.
DISCUSSION

In the theoretical calculation, the bearing capacity factors $N_c$, $N_q$, and $N_t$ were obtained for different $\phi$ values and various semi angles $\beta$ for perfectly smooth and rough bases. The theoretical values $N_c$, $N_q$, and $N_t$ are presented in Figs. 2(a), (b), (c) respectively for the perfectly smooth and rough wedges.

The factors for smooth wedge decrease rapidly with decreasing semi angle $\beta$. However for $\beta < 30^\circ$ approximately, the factor $N_t$ increased again. For rough wedge the factors are sensibly unaffected by the wedge angle except for $\beta$ about $< 30^\circ$ when the factors increased rapidly with further decrease in $\beta$ values. The test results showing the load settlement curves for a wedge of $\beta = 30^\circ$ on loose, medium and dense sands are present-

Fig. 5 Comparison between theoretical and experimental values of bearing capacity factor $N_t$.
ed in Figs.3(a), (b), (c), respectively. Similar test results for wedges of $\beta = 15^\circ, 30^\circ, 45^\circ$ and $90^\circ$ on clay are presented in Figs.4(a), (b), (c) and (d), respectively.

The results indicate (Figs.5(a), (b), (c)) that the values of bearing capacity factor $N_c$ for smooth wedge decrease rapidly with decreasing semi angle $\beta$, but for $\beta < 30^\circ$ approximately, the factor $N_c$ increased again. For rough wedge the values of $N_c$, are sensibly unaffected by $\beta$ except for $\beta < 30^\circ$ approximately, when the factors increased rapidly with further decrease in the angle $\beta$.

In the case of clay, the $N_c$ values for smooth wedge decreased continuously with decreasing semi angle $\beta$ (Fig.6). In the case of rough wedge, $N_c$ values are reasonably constant till $\beta = 30^\circ$ and then increased with further decrease in the values of $\beta$. The observed values of $N_c$ and $N_t$ are seen to agree closely with the computed values. From both theoretical analyses and experimental observations, the factors for smooth wedges are seen to be smaller than those for rough wedges.

**CONCLUSIONS**

An analytical investigation using two dimensional theory of bearing capacity of wedge shaped foundation has been made on the basis of limit equilibrium. The theoretical results are presented as bearing capacity factors for different angles of internal friction $\phi$, semi angle $\beta$, for both smooth and rough surfaces. Model tests were carried out with smooth and rough footings of various semi angles of wedge, on clay and sand under axial loads.

The results of these tests were analyzed according to the general bearing capacity equation to determine the bearing capacity factors. The theoretical values of the bearing capacity factors $N_c$, $N_q$, $N_t$ are compared with the results of the experiments. It is concluded that the experimental values of bearing capacity factors agree closely with the theoretical values.

The present theory is suitable for obtaining the bearing capacity factors $N_c$ and $N_t$ for a given semi angle $\beta$ and roughness of the wedge.

**References**

均一地盤表面における楔型基礎の支持力

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摘要
種々の先端半角βおよび粗度を有する楔型基礎の均一地盤表面における鉛直支持力問題を塑性論的に解析した。地盤はMohr-Coulombの破壊基準に従う完全塑性体と考え、すべり線解法を用いて解析を行い、楔支持力係数, $N_a$, $N_q$, $N_t$を地盤の内部摩擦係数$\phi$, 先端半角βおよび粗度の関数として表した。

砂地盤および粘土地盤において、種々のβおよび粗度を有する楔型基礎の支持力実験を行った。実験は乾燥した豊浦砂および飽和した有明粘土を用いた。実験結果より支持力係数の実験値を求め、計算値と対比した結果両者はかなりよく一致することが明らかとなった。

キーワード：支持力，粘土，砂，モデル試験，浅い基礎